Mathematicians in France and in the world

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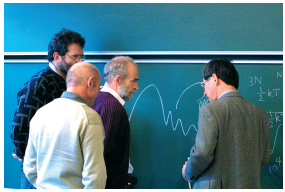


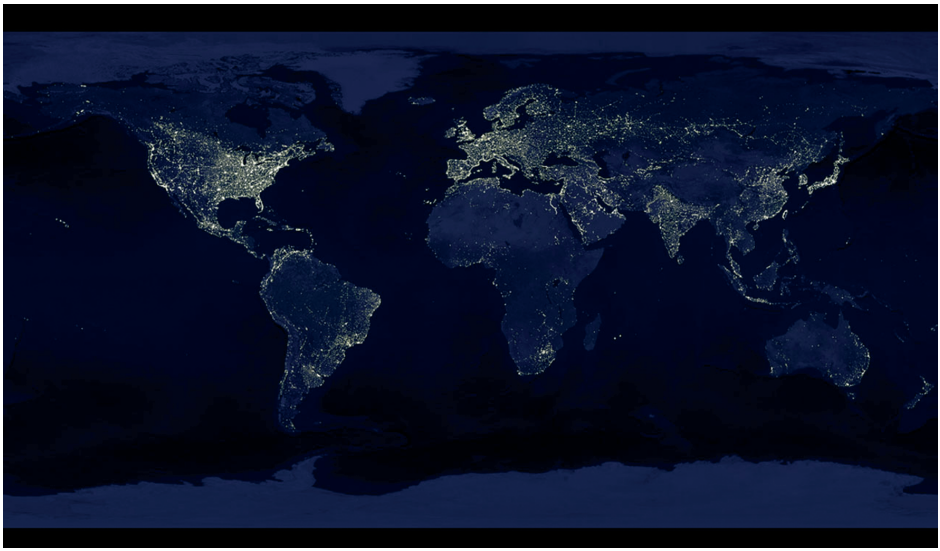
At the end of the 19th century, there were very few ``geometers'', as mathematicians were formerly called. In one century, their numbers have augmented considerably. Today, they are facing profound changes in their discipline.

During the course of the 20th century, the mathematical community has undergone a major expansion. From a few hundred members in 1900, it has passed to tens of thousand (probably about 80000) members 100 years later. To make an estimate of this kind it is necessary at first to agree upon a definition of the term ``mathematician''. We reserve this term for those who have reached the education level equivalent to the doctoral thesis and whose profession attaches importance to mathematical research or to the assimilation of its results.

This choice may be considered to be a bit restrictive because it leads us, for example, to exclude from our field of vision almost all teachers in secondary schools - a category whose numbers have also increased considerably in all countries of the world during the second half of the 20th century.

This growth is the result of several simultaneous processes. First of all, there was the realisation, just after the Second World War, of the importance of the sciences for economic and industrial development. In addition, new groups of people have been able to enter the profession. Such is the case for women, although there are great disparities between different countries. But at the same time, an academic community bringing together the participants in higher education made its appearance in almost every country. To give only one example, the first mathematicians from sub-Saharan Africa worked for their doctorate in a university in a Western country or in the Soviet Union.

The next generation often pursued their studies in their own country: in the decade 1990-2000, many countries of sub-Saharan Africa set up autonomous establishments of higher education and gained their independence from this point of view. In the coming years, the expansion will continue with probably a considerable reinforcement of the mathematical communities, in other countries such as China and India..



The earth viewed at night. The world distribution of nocturnal lights is not without reminding us of the centres of mathematical activity. However, not all mathematicians work at night! (Photo C. Mayhew and R. Simmon/NASA-GSFC)

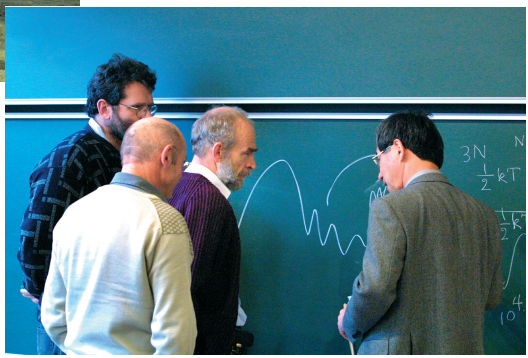
A community of researchers and its network of learned societies

How are mathematical communities organised? The expansion of the international mathematical community was accompanied by an organisation stimulated by learned societies, almost all of which survive thanks to the devotion and the involvement of volunteers. Mathematical societies are still quite small in size, except for the American Mathematical Society, which has nearly 15000 members and more than 200 employees.

The first stage occurred on the national level, usually at a time when the authorities realised that the development of the sciences could represent an economic and a military asset. Thus the *Société mathématique de France* (SMF), just as the *Société française de physique*, was founded in 1872, immediately after the 1870 defeat by Germany and the consequent reflection on its causes. This narrow nationalist perspective is fortunately forgotten

The International Mathematical Union was created in 1896. It continues to be small. Its main responsibility is to provide the framework for organising the International Congress of Mathematicians, a four-yearly event which has become an unavoidable rendez-vous for the mathematical community on a global scale. Its executive committee also undertakes to nominate the commission which awards the Fields medals every four years; they represent the most prestigious award in mathematics as there is no Nobel Prize in this discipline.

The end of the 20th century saw the emergence of intermediate structures at the level of continents. The example was given by our African colleagues who created the African Mathematical Union as early as the 1980's. Then came the European Mathematical Society



The IHÉS (Institut des hautes études scientifiques), at Bures-sur-Yvette in the suburbs of Paris, and a discussion between mathematicians in its buildings. The IHÉS, devoted to fundamental mathematics and to theoretical physics, is a prestigious research institute. It has only 7 permanent professors but welcomes some 200 researchers of different nationalities each year, for variable durations. Recently, some of its mathematicians have started to concentrate on problems related to molecular biology. (Photo IHÉS and IHÉS-Outsider Agency)

(EMS), whose gestation was long - as for the European Union - and which brings together all the national societies form geographical Europe and from Israel, and the UMALCA, which unites the mathematicians from the Caribbean and from South America. These new structures were born of the desire to reinforce collaborations on the scale of a sub-continent and to have a representative interlocutor in the face of the appearance of a new political level (as in Europe) or to control the draining of resources by North America (as was the case for South America) following the painful period of military dictatorships

An increasingly broad presence in industry and the services

Where are mathematicians employed? The great innovation is that, nowadays, mathematicians are present in many sectors of industry and the service industry. There is, however, no ``mathematical industry'', such as a chemical industry or a pharmaceutical industry. Indeed, the jobs entrusted to people with high mathematical competence often carry very different names, which makes it difficult to count the number of ``industrial mathematicians''. A recent estimate leads one to think that there are nearly 2000 of them employed this way in France. This number is to be compared with the number of their academic counterparts (mathematicians in universities and those working in various research organisations) which can be estimated much more reliably as approximately 4000. The division of this academic community between public research organisations and higher education (approximately 10% against 90%) is a little singular: generally, in other scientific disciplines, choices are different and a much more important proportion devote all their time to research, without teaching duties. Which sectors are particularly interested in employing mathematicians? Banks and insurance companies make an increasingly intensive use of mathematical competence; the products which they sell often rely upon a mathematical construction which is at their very base. But it is the same with a certain number of high-technology companies in which the study of complex systems requires

a mathematical approach made possible by the powerful computational tools provided by the new generation of computers. These new openings are of a kind which might considerably change the image of mathematics among students. However, they have not yet been completely assimilated by French higher education; the most frequent reason is the excessive inertia of the education system, which remains centred around training for the academic professions.

Mathematicians are confronted with a new situation

These new developments have not been without effect on the the way mathematics is organised, as much in the establishments of higher education and research as at the level of publications. The resulting situation has sometimes been presented as a battle between ``pure mathematics'' and ``applied mathematics''. This way of seeing things is unjustified for at least two reasons. On one hand, historical examples abound of situations where new mathematics was created at the behest of external demand; on the other hand, the new fields to be conquered cannot be approached by declaring a priori which part of mathematics will be the key to the solution of the problems which arise. Many surprise-connections can be noted, which proves that the pure/applied dichotomy is unproductive in the final analysis. It was in this context of internal tension in the mathematical community that the *Société de mathématiques appliquées et industrielles* (SMAI) was born in France in 1983. Twenty years later, the two societies, SMF and SMAI, have found a way of effective co-operation and together carry out programmes of common interest. They count more than 3000 members between them, of which many in the SMAI are from well outside the academic community The principal innovation comes from the possibility of studying more and more complex systems thanks to the use of models of various kinds. Modelling is today a method to which one often resorts. This new passion requires a thorough reflection on the foundations, including philosophical foundations, of this approach. One of the capabilities, which it is advisable to develop, is the confrontation of the model with the reality which it is supposed to represent. One can nevertheless underline two major tendencies which feed on these new contacts between mathematics and a world which is external to it: a renewal of interest in finite structures (mathematical structures involving only a finite number of elements) and the ubiquity of stochastic approaches (involving random processes). In the latter field, France has taken a remarkable turn, compared with the situation in other countries with the same level of development, except perhaps for the underrepresentation of statistics and of data analysis. On the other hand, the teaching of discrete mathematics, i.e., having to do with finite structures, remains quite discrete in France: very few higher education course offer a sufficiently complete training in this field Recently, on the occasion of a conference devoted to the history of geometry in the second half of the 20th century, Stephen Smale, an American mathematician who is one of the fathers of modern topology and who has since taken a great interest in numerical analysis, made a pertinent remark: the extraordinary growth of mathematics today is often due to people whom mathematicians tend not to consider as belonging to their community.

It is true that statistics, cybernetics, operations research, control theory are often

poorly represented in university mathematics departments, whereas the heart of all these disciplines is really mathematical. One could say the same thing about a good part of theoretical computer science: the depth and the force of the organic links it maintains with mathematics are not always appreciated by the mathematicians themselves. This situation opens up the possibility of considerable growth of the mathematicial community provided that they are less prompt at excluding these new activities from their field. With greater curiosity and openmindedness there will be greater stimulation and new spheres of activity, for the greater good of the development of mathematics itself.

The changes in the profession require new training programmes

One of the first things to be recognised is related to the practice of the mathematician's profession required by these new contacts, a practice which cannot limit itself to proving theorems.

The pressing need is that a sufficient number of mathematicians with very different backgrounds get interested in applications. This requires them learning to talk to specialists in other disciplines, and listening well.

Already one notes the introduction of specialised training, in financial mathematics for example, in various higher education structures throughout the world. Other fields, for which important openings outside the academic world have appeared, will certainly see the light of day, on a scale adapted to these openings; it is already the case as in regard to the training of actuaries, and it can be foreseen that mixed training programmes will be introduced at the interface of mathematics with biology and medicine, for example

But allowing too-specialised training programmes to proliferate would be an error for two reasons: the narrowness of an approach of this kind on the one hand, and, on the other hand, the risk of schism in the mathematical community that such a practice would present. In order that students perceive in a more natural way the new orientations accessible to mathematical methods, major extensive modifications of the teaching curricula will probably have to be put into place. One must create a fluidity between the academic world and the world of industry and the service industry; this is a precondition for the generation of good problems, generally dealing with new fields, so that rather spontaneously these problems will be dealt with with the necessary level of depth.

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